Describing the ADD model in a warped geometry

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We propose a new description of the $(4+N)$-dimensional Arkani-Hamed-Dimopoulos-Dvali (ADD) model in a $(4+1)$-dimensional warped geometry to solve the gauge hierarchy problem. It has the same KK spectrum as in the ADD model and recovers its phenomenons that do not involve the interaction among the graviton KK modes. There is no hierarchy between the fundamental length and the size of the extra dimension. An explicit realization is constructed in the nonlocal gravity theory to give a warped description of the six-dimensional ADD model. Remarkably, the equivalent number N of the extra dimensions in this description may be non-integral, which provides a new mechanism to escape the experimental constrains.

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I. INTRODUCTION

The idea of extra dimensions opens a new way to solve the gauge hierarchy problem—the large difference between the electroweak scale $M_{\rm EW} \sim 1$TeV and the Planck scale $M_{\rm Pl} \sim 10^{16}$TeV. One of the famous models is the Arkani-Hamed-Dimopoulos-Dvali (ADD) model [1, 2], also called the model of large extra dimensions. In this model, the fundamental scale is $M_* \sim 1$TeV, and the hierarchy between the effective Planck scale $M_{\rm Pl}$ and the fundamental scale M_* is generated by the large volume of the extra dimensions [1]:

$$M_{\rm Pl}^2 \sim M_*^{N+2} R^N, \tag{1}$$

where R and N are the size and number of the extra dimensions, respectively. In this theory, the standard model particles are assumed to be confined on the brane and the extra dimensions are flat, so the electroweak scale is the same as the fundamental scale: $M_{\rm EW} \sim M_*$. One of the famous predictions of the ADD model is that it may break down the gravitational inverse-square law in experimentally accessible regions, since the spacing of graviton KK modes is determined by the size of large extra dimensions R: $\Delta E \sim R^{-1}$. However, a remained hierarchy between the size of the extra dimensions and the fundamental length M_*^{-1} is introduced as $R \sim 10^{32/N} {M_*}^{-1}$. Furthermore, there is no brane tension in the model.

A good inspiration to solve the remained hierarchy problem in the ADD model comes from the Randall-Sundrum 1 (RS1) model [3] in five dimensions with the warped geometry given by

$$\begin{aligned} ds^2 &= e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 && (2)\\ &= \frac{1}{(1+k|z|)^2}\big(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\big), && (3)\end{aligned}$$

where k is the AdS curvature scale, $y \in [-y_b, y_b]$ and $z \in [-z_b, z_b]$ with $kz_b = e^{ky_b} - 1$ are the physical and conformal coordinates of the fifth dimension, respectively. The effective Planck and electroweak scales read [3]

$$M_{\rm Pl}^2 = \frac{M_*^3}{k}(1-e^{-2ky_b}), \quad M_{\rm EW} = M_* e^{-ky_b}, \tag{4}$$

where the fundamental scale is assumed as $M_* \sim k \sim 10^{16}$TeV. The gauge hierarchy problem is solved by a small physical length of the fifth dimension $y_b \sim 37k^{-1} \sim 37M_*^{-1}$. From another point of view, a large conformal size of the extra dimension generates the hierarchy between the Planck scale and the electroweak scale via

$$M_{\rm Pl} \sim M_{\rm EW}(M_* z_b). \tag{5}$$

Furthermore, the spacing of graviton KK modes is directly determined by the conformal size z_b: $\Delta E \sim z_b^{-1} \sim 1$TeV, rather than the physical size y_b [3]. That is to say, in warped extra dimension model, the new gravitational effect of extra dimension appears at the conformal scale z_b but not the physical scale y_b. However, the conformal scale $z_b \sim 1\text{TeV}^{-1}$ is too small in RS1 model.

In this letter, we would like to build new warped brane models that solve the remained hierarchy problem in the

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ADD model and does not change its phenomenons. The new model would have a large conformal size to recover the phenomenons of ADD model and still keep a small physical size to solve the remained hierarchy problem. Further, in order to maintain the fundamental scale in the ADD model $M_* \sim M_{\rm EW} \sim 1$TeV, the standard model particles should be confined at $y = 0$ since the mass parameters of fields confined at $y = y_0$ in the background (2) has a redshift e^{-ky_0} [3, 4]. Thus, the graviton zero mode should be localized near $y = y_b$ to generate a large Planck mass in order to solve the gauge hierarchy problem. This is opposite to the RS1 model constructed in general relativity, in which the graviton zero mode is localized near $y = 0$. We will realize the special property of the graviton zero mode and build the model in the nonlocal gravity theory, which is one of the simplest realization of this class of models. In fact, a special model was constructed in the scalar-tensor gravity in Ref. [5]. However, there is no real brane with tension constructed by matter. We would like to construct a brane with tension. This is also one motivation of this letter.

Different from the usual way to modify Einstein gravity by local generalization, Deser and Woodard developed a modified gravity theory by adding a nonlocal term [6, 7]. This nonlocal gravity theory was proposed to explain the cosmic acceleration without the cosmological constant [8–12]. It was found that this theory possesses the same gravitational degrees of freedom and initial value constraints as general relativity, and has no ghost graviton mode [13].

We will construct a five-dimensional warped brane model in the nonlocal gravity. It will solve the remained hierarchy and recover the phenomenons of the six-dimensional ADD model that do not involve the interaction between the graviton KK modes. We will also propose a possible way to construct a class of generalized five-dimensional warped models that describe the $(4+N)$-dimensional ADD model.

II. THE MODEL

We start with the following action of the nonlocal gravity theory proposed in Ref. [6] in $(d+1)$-dimensional spacetime:

$$S = \frac{M_*^{d-1}}{2}\int d^{d+1}x\sqrt{-g}\,\mathcal{R}\Big(1+f(\Box^{-1}\mathcal{R})\Big) + S_{\tt b}, \quad (6)$$

where M_* is the fundamental $(d+1)$-dimensional Planck mass, $\mathcal{R}$ is the curvature scalar, $\Box^{-1}$ is the inverse of the d'Alembertian operator, and $S_{\tt b}$ is the action describing the brane configuration. Note that in order to define this theory, we should specify what definition of $\Box^{-1}$ we use. This model has a local form by adding two auxiliary fields η and ξ [10, 11, 14]:

$$S = \frac{M_*^{d-1}}{2}\int d^{d+1}x\sqrt{-g}\,(\psi\mathcal{R} + \xi\Box\eta) + S_{\tt b}, \quad (7)$$

where we have defined $\psi \equiv 1 - f(\eta) + \xi$. To ensure the stability, we should require $\psi > 0$. By varying the action (7) with respect to ξ and η respectively, one easily gets

$$\Box\eta = \mathcal{R}, \quad \Box\xi = -\mathcal{R}f_\eta, \quad (8)$$

where $f_\eta \equiv \frac{df}{d\eta}$. Now the choice of the definition of $\Box^{-1}$ becomes the choice of the solution of $\Box\eta = \mathcal{R}$. Once we have chosen a specific solution of η, the original nonlocal theory is defined. It means that different solutions of η correspond to different nonlocal theories, rather than different solutions of one nonlocal theory.

The modified Einstein equations are given by

$$\begin{aligned} &\psi\mathcal{R}_{AB} - \frac{1}{2}\left(\psi\mathcal{R} - \partial_C\xi\partial^C\eta\right)g_{AB} \\ +&\ (g_{AB}\Box - \nabla_A\nabla_B)(1-\psi) - \frac{1}{2}\big(\partial_A\xi\partial_B\eta + \partial_A\eta\partial_B\xi\big) \\ =&\ M_*^{1-d}T^{(\tt b)}_{AB}, \end{aligned} \quad (9)$$

where $A, B, C\cdots$ denote the bulk indices $0, 1, 2, \cdots, d$, and $T^{(\tt b)}_{AB}$ represents the energy-momentum tensor of the brane.

The metric for the flat brane world model is assumed as [3]

$$ds^2 = a^2(y)\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \quad (10)$$

where the brane coordinate indices $\mu, \nu = 0, 1, \cdots, d-1$ and $\eta_{\mu\nu} = \texttt{diag}(-, +, \cdots, +)$. We consider an S_1/Z_2 orbifold extra dimension as in Ref. [3]; so the physical extra dimensional coordinate $y \in [-y_b, y_b]$. The action describing one brane with tension σ_+ located at $y = 0$ and another brane with tension σ_- located at $y = y_b$ is given by

$$S_{\tt b} = \int d^dx\Big[\sqrt{-q(y=0)}\,\sigma_+ + \sqrt{-q(y=y_b)}\,\sigma_-\Big], \quad (11)$$

where $q_{\mu\nu}(y=0)$ and $q_{\mu\nu}(y=y_b)$ are the induced metrics at $y=0$ and $y=y_b$, respectively.

Then the $\mu\nu$ and dd components of the modified Einstein equations (9) and the equations (8) for η and ξ are given by

$$\begin{aligned} &\psi'' + (d-1)H\psi' + \left[(d-1)H' + \frac{1}{2}d(d-1)H^2\right]\psi \\ &\quad + \frac{1}{2}\xi'\eta' - M_*^{1-d}\big[\sigma_+\delta(y) + \sigma_-\delta(y-y_b)\big] = 0, \quad (12) \\ &\frac{1}{2}\xi'\eta' - dH\psi' - \frac{1}{2}d(d-1)H^2\psi = 0, \quad (13) \end{aligned}$$

and

$$\begin{aligned} \eta'' + dH\eta' &= -2dH' - d(d+1)H^2, \quad (14) \\ \xi'' + dH\xi' &= [2dH' + d(d+1)H^2]f_\eta, \quad (15) \end{aligned}$$

respectively, where $H \equiv a'/a$ and the prime denotes the derivative with respect to the coordinate y. Subtracting (13) from (12) we get

$$\begin{aligned} &\psi'' + (2d-1)H\psi' + \big[(d-1)H' + d(d-1)H^2\big]\psi \\ =&\ M_*^{1-d}\big[\sigma_+\delta(y) + \sigma_-\delta(y-y_b)\big], \end{aligned} \quad (16)$$

which can be rewritten as

$$\begin{aligned}&\Big(\partial_y + dH\Big)\Big(\partial_y + (d-1)H\Big)\psi \\ &= M_*^{1-d}\big[\sigma_+\delta(y) + \sigma_-\delta(y-y_b)\big]. \quad (17)\end{aligned}$$

Now it can be checked that the system supports the following solution for the warp factor $a(y)$ and the function $\psi(y)$:

$$a(y) = e^{-k|y|}, \quad \psi = c_1 e^{dk|y|} + c_2 e^{(d-1)k|y|}. \quad (18)$$

So the RS1 geometry is reproduced. The condition $\psi > 0$ becomes $c_2 > -c_1 e^{k|y|}$. After integrating Eq. (17) at $y = 0$ and $y = y_b$ respectively, we can obtain the relations

$$c_1 = \frac{\sigma_+}{2kM_*^{d-1}}, \quad \sigma_- = -\sigma_+ e^{dky_b}. \quad (19)$$

We assume that the brane located at $y = 0$ has positive tension, i.e., $\sigma_+ > 0$. Then the brane located at $y = y_b$ is a negative tension brane. The relation between the σ_+ and σ_- is different from the case in the RS1 model, where $\sigma_+ = -\sigma_-$. It is interesting to consider the tensionless limit $\sigma_+ = 0$. In this limit $c_1 = 0$ and $\psi \propto e^{(d-1)k|y|}$. In the process similar to the following, we will find it has the same spectrum as in the $(d+1)$-dimensional ADD model with codimension one.

The solutions of the auxiliary fields η and ξ are somewhat complicated. We will use Eq. (14) for η as an example. We denote the region $y \in [ny_b, (n+1)y_b]$ as the n-th section in the region $y \in [0, +\infty)$ and denote the solution of the field $\eta(y)$ in this section as $\eta_n(y)$. Then the solution of Eq. (14) is given by

$$\eta_n(y) = (-1)^n(d+1)ky + \lambda_n e^{(-1)^n dk(y-ny_b)} + \rho_n, \quad (20)$$

where the coefficients λ_n are determined by the recursion relation $\lambda_1 = 2(d-1)/d - e^{dky_b}\lambda_0$, $\lambda_{n+2} = \big(1 - e^{-(-1)^n dky_b}\big)2(d-1)/d + \lambda_n$. The coefficients ρ_n can be determined by the continuity condition of $\eta(y)$. The exact solutions of the auxiliary fields η and ξ are unnecessary since they do not influence the spectrum of the massive KK gravitons and the couplings of these gravitons to matter.

III. PHYSICAL IMPLICATIONS

Since the metric (10) has the d-dimensional Poincare symmetry, we can consider the transverse and traceless (TT) part of the perturbation separately, which corresponds to the spin-2 graviton in d dimensions. The tensor perturbation is parameterized as

$$ds^2 = a^2(\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu + dy^2, \quad (21)$$

where the perturbation $h_{\mu\nu}(x, y)$ satisfies the TT conditions $\partial^\mu h_{\mu\nu} = 0 = \eta^{\mu\nu}h_{\mu\nu}$. The $\mu\nu$ component of the linearized perturbation equations reads

$$a^2 h''_{\mu\nu} + \Big(daa' + a^2\frac{\psi'}{\psi}\Big)h'_{\mu\nu} + \Box^{(d)}h_{\mu\nu} = 0, \quad (22)$$

where $\Box^{(d)}$ denotes the d-dimensional d'Alembertian operator. After changing to the conformal coordinate z with $adz = dy$ and defining $A \equiv a^{d-1}\psi$, the linearized equation (22) becomes

$$\partial_z\partial_z h_{\mu\nu} + \frac{\partial_z A}{A}\partial_z h_{\mu\nu} + \Box^{(d)}h_{\mu\nu} = 0. \quad (23)$$

We decompose the tensor perturbation $h_{\mu\nu}$ as

$$h_{\mu\nu}(x, z) = \varepsilon_{\mu\nu}(x)A^{-\frac{1}{2}}(z)\Psi(z), \quad (24)$$

where the four-dimensional part of the graviton KK mode $\varepsilon_{\mu\nu}(x)$ satisfies $\Box^{(d)}\varepsilon_{\mu\nu}(x) = m^2\varepsilon_{\mu\nu}(x)$. Then we obtain a Schrödinger-like equation for the extra dimensional part:

$$-\partial_z\partial_z\Psi + \left(\frac{1}{2}\frac{\partial_z\partial_z A}{A} - \frac{1}{4}\frac{(\partial_z A)^2}{A^2}\right)\Psi = m^2\Psi. \quad (25)$$

It can be rewritten as $\big(\partial_z + \frac{1}{2}\frac{\partial_z A}{A}\big)\big(-\partial_z + \frac{1}{2}\frac{\partial_z A}{A}\big)\Psi = m^2\Psi$, which implies that the eigenvalues m^2 are nonnegative and the brane system is stable under the tensor perturbation.

From now on we restrict our discussion in the case of $d = 4$. Setting $m^2 = 0$ in Eq. (25), we can obtain the normalized zero mode

$$\Psi_0(z) = \frac{A^{\frac{1}{2}}}{N_0} = \left(\frac{\frac{\sigma_+}{2kM_*^3}(1 + k|z|) + c_2}{\frac{\sigma_+}{2kM_*^3}(2z_b + kz_b^2) + 2c_2 z_b}\right)^{\frac{1}{2}}. \quad (26)$$

Even though the RS1 background is reproduced, the graviton zero mode is localized near the negative tension brane, rather than near the positive one in the RS1 model [3]. So in our model we should confine the standard model particles on the positive tension brane to solve the gauge hierarchy problem. Because the graviton zero mode is localized near the negative tension brane, we need the compactification of the extra dimension to recover the effective four-dimensional gravity.

Substituting the zero mode into the action (7) and making the reduction, we get the effective four-dimensional Planck mass:

$$M_{\rm Pl}^2 = M_*^3\int dz a^3\psi = M_*^3 z_b\left[\frac{\sigma_+}{2kM_*^3}(2 + kz_b) + 2c_2\right]. \quad (27)$$

Because $a(0) = 1$, the vacuum expectation value of the Higgs scalar, which sets all mass parameters of the standard model, is not warped down or redshifted. So all the mass scales are not suppressed on the positive tension brane and the electroweak scale remains the fundamental scale $M_{\rm EW} \sim M_*$ [3]. In order to solve the gauge hierarchy problem, we set the dimensionless parameter c_2 as $c_2 \sim 1$ and all the fundamental scales as 1TeV, which means we set M_* and k as 1TeV and σ_+ as $(1\text{TeV})^4$. Then the effective four-dimensional Planck mass $M_{\rm Pl} \sim 10^{16}$TeV leads to the condition $kz_b \sim 10^{16}$, which corresponds to $ky_b \sim 37$ (or $y_b \sim 37\text{TeV}^{-1}$) in the physical coordinate.

In the following discussion, we analyze the spectrum of the massive KK modes and their couplings to the matter confined on the positive tension brane. With the Neumann boundary condition at the positive tension brane: $\partial_z h_{\mu\nu}|_{z=0} = 0$, we can obtain the solutions for the Schrödinger-like equation (25):

$$\Psi_n(z) = \frac{1}{N_n}\left(z+\frac{1}{\beta}\right)^{\frac{1}{2}}\left[J_0\Big(m_n(z+\frac{1}{\beta})\Big) + \alpha Y_0\Big(m_n(z+\frac{1}{\beta})\Big)\right], \tag{28}$$

where $\frac{1}{\beta} \equiv \frac{1}{k} + \frac{2M_*^3 c_2}{\sigma_+}$, $\alpha = -\frac{J_1(\frac{m_n}{\beta})}{Y_1(\frac{m_n}{\beta})}$, and N_n is the normalization constant. The spectrum m_n of the graviton KK modes can be obtained by the Neumann boundary condition at z_b: $\partial_z h_{\mu\nu}|_{z=z_b} = 0$, which reads as

$$\frac{J_1\left(m_n(z_b+\frac{1}{\beta})\right)}{J_1(m_n/\beta)} = \frac{Y_1\left(m_n(z_b+\frac{1}{\beta})\right)}{Y_1(m_n/\beta)}. \tag{29}$$

Since $kz_b \sim 10^{16}$ and $c_2 \sim 1$, the first few KK modes satisfy the condition $\frac{m_n}{\beta} \ll 1$ and $\alpha \approx \frac{\pi}{8}(\frac{m_n}{\beta})^2 \to 0$. Then the solution (28) becomes

$$\Psi_n(z) \approx \frac{1}{N_n}\left(z+\frac{1}{\beta}\right)^{\frac{1}{2}} J_0\left(m_n(z+\frac{1}{\beta})\right). \tag{30}$$

In this approximation, the spectrum reads

$$m_n \approx \frac{x_n}{z_b}, \tag{31}$$

where x_n satisfies $J_1(x_n) = 0$, and $x_1 = 3.83$, $x_2 = 7.02$, $x_3 = 10.17$, $\cdots$. The normalization constant N_n can be determined as follows:

$$1 = \int_{-z_b}^{z_b} \Psi_n^2 dz \approx \frac{2}{N_n^2}\int_0^{z_b} z\,[J_0(m_n z)]^2\,dz \approx \frac{1}{N_n^2} z_b^2 [J_0(m_n z_b)]^2. \tag{32}$$

Using the approximation $J_0(x) \approx \sqrt{\frac{2}{\pi x}}\cos(x - \frac{1}{4}\pi)$ and the approximate formula of the zero point of $J_1(x_n)$, $x_n \approx (n+\frac{1}{4})\pi$, we have $N_n \approx \sqrt{\frac{2z_b^2}{\pi x_n}}$. Then the normalized KK modes are

$$\Psi_n(z) \approx \sqrt{\frac{\pi x_n}{2z_b^2}}\left(z+\frac{1}{\beta}\right)^{\frac{1}{2}} J_0\left(m_n(z+\frac{1}{\beta})\right). \tag{33}$$

The interaction between these KK modes and the matter confined on the positive tension brane is

$$L_{\rm int}^{(n)} = M_*^{-\frac{3}{2}}\widetilde{T}^{\mu\nu}(x)h_{\mu\nu}^{(n)}(x,0) = \widetilde{\xi}^{(n)}\widetilde{T}^{\mu\nu}(x)\varepsilon_{\mu\nu}^{(n)}(x), \tag{34}$$

where $\widetilde{T}_{\mu\nu}$ is the symmetric conserved Minkowski space energy-momentum tensor. Then it is simple to get

$$\widetilde{\xi}^{(n)} \propto M_*^{-\frac{3}{2}}\sqrt{\frac{x_n}{\beta z_b^2}} \propto \frac{x_n^{\frac{1}{2}}}{10^{16}\rm TeV}. \tag{35}$$

Equation (35) shows that the couplings of the first few KK modes to the matter have the similar strength to the coupling of the massless graviton to the matter. However, the higher KK modes have stronger couplings to the matter. To estimate their effects, we first consider the process that involves emission of gravitons, which could be observed as missing energy [1, 2, 15]. The total cross section for this process is of order

$$\Sigma_n(\widetilde{\xi}^{(n)})^2 = \frac{1}{E^2}\frac{E^4}{(1\rm TeV)^4}. \tag{36}$$

So the cross section would have a significant increase when the energy approach 1TeV. On the other hand, the massive KK modes with the mass gap 10^{-4}eV would contribute an obvious deviation to Newton potential when distance less than the critical distance $R_c \sim 2$mm.

The total cross section (36) and the critical distance are the same as in the six-dimensional ADD model. The mechanism can be clearly seen from Eq. (27), in our five-dimensional warped model $M_{\rm Pl}^2 \sim M_*^3 k z_b^2 \sim M_*^4 z_b^2$ and in the six-dimensional ADD model $M_{\rm Pl}^2 \sim M_*^4 R^2$, which implies $z_b \sim R$.

Further, in the six-dimensional ADD model, the number of KK modes with same mass m_n is about $m_n R$ and they have the same couplings of $\frac{1}{M_{\rm Pl}}$ to matter. In our five-dimensional warped model, there is only one KK mode with mass m_n and the coupling to matter is $\frac{(m_n z_b)^{\frac{1}{2}}}{M_{\rm Pl}}$. Because $m_n R(\frac{1}{M_{\rm Pl}})^2 \sim (\frac{(m_n z_b)^{\frac{1}{2}}}{M_{\rm Pl}})^2$, the sum of the propagators of all the KK modes in the ADD model is equal to the propagator with the same mass in our five-dimensional model. So the phenomenons that do not involve the interaction among the KK modes are the same in these two models.

IV. THE WARPED DESCRIPTION OF THE $(4+N)$-DIMENSIONAL ADD MODEL

In this section, we point out a possible way to further generalize the five-dimensional warped model to describe the $(4+N)$-dimensional ADD model. We also assume that the fundamental scale is $M_* \sim 1$TeV and the standard model particles are confined on the brane at $y = 0$. A possible class of actions in five dimensions are

$$S = \frac{M_*^3}{2}\int d^5x\sqrt{-g}\Big[\psi\mathcal{R} + \mathcal{L}(g_{AB},\psi,\Phi_i)\Big] + S_{\rm b}, \tag{37}$$

where only the field ψ nonminimally couples to the curvature scalar, and Φ_i in the Lagrange density $\mathcal{L}$ denote other dynamic and/or nondynamic fields. Assume that the system supports a family of solutions:

$$a(y) = e^{-k|y|}, \quad \psi(y) \propto e^{\alpha k|y|}, \tag{38}$$

where $k \sim M_*$ and the value of the dimensionless parameter α is determined by the detail of the Lagrange density. For our model (7), $\alpha = 4$.

The graviton zero mode can be written as $\Psi_0 \propto a^{3/2}\psi^{1/2} \propto e^{\frac{1}{2}(\alpha-3)k|y|} = (1+k|z|)^{(\alpha-3)/2}$, which is localized near $y = y_b$ and $y = 0$ when $\alpha > 3$ and $\alpha < 3$, respectively. The effective four-dimensional Planck mass is determined by

$$M_{\rm Pl}^2 \sim M_*^3 (kz_b)^{\alpha-3} z_b. \tag{39}$$

Comparing with the corresponding relation in the $(4+N)$-dimensional ADD model, $M_{\rm Pl}^2 \sim M_*^{N+2} R^N$, if $\alpha = N+2$, then $z_b \sim R$ and it can be shown that the two models will share the same KK spectrum. Note that the physical size of the extra dimension is $y_b \sim \frac{74}{N} M_*^{-1}$, which indicates that the remained hierarchy problem in the ADD model is solved in this generalized warped description. In the following, we consider the case of $N \geq 2$.

Having reproduced the graviton KK spectrum in the ADD model, we turn to the question of the couplings between the graviton KK modes and the matter confined on the brane at $y = 0$. With the similar procedures as in section III, we can write the graviton KK modes as

$$\Psi_n(z) \approx \frac{1}{N_n}\left(z+\frac{1}{k}\right)^{\frac{1}{2}} J_{\frac{N}{2}-1}\left(m_n(z+\frac{1}{k})\right). \tag{40}$$

The spectrum is given by $m_n \approx \frac{x_n}{z_b}$, where x_n are determined by $J_{\frac{N}{2}}(x_n) = 0$. The normalization constants are $N_n \propto z_b/\sqrt{x_n}$ and then $\Psi_n(0) \propto x_n^{\frac{N-1}{2}} k^{\frac{1-N}{2}} z_b^{-\frac{N}{2}}$. The couplings of these KK modes to the matter at $y = 0$ are given by

$$\widetilde{\xi}^{(n)} \propto M_*^{-\frac{3}{2}} \Psi_n(0) \propto M_{\rm Pl}^{-1} x_n^{\frac{N-1}{2}}. \tag{41}$$

In the $(4+N)$-dimensional ADD model, the number of the graviton KK modes with the same mass m_n is about $m_n^{N-1} R^{N-1}$ and each mode has a coupling of $M_{\rm Pl}^{-1}$ to the matter. In the warped description, there is only one graviton KK mode with mass m_n and its coupling to matter is about $M_{\rm Pl}^{-1}(m_n z_b)^{\frac{N-1}{2}}$. Because $m_n^{N-1} R^{N-1} (M_{\rm Pl}^{-1})^2 = (M_{\rm Pl}^{-1}(m_n z_b)^{\frac{N-1}{2}})^2$, the sum of the propagators of all the graviton KK modes with mass m_n in the ADD model is equal to the propagator with the same mass in the five-dimensional warped description. So our description recovers the phenomenons of the ADD model that do not involve the interaction among the graviton KK modes.

This class of models give a new realization of the ADD model: the effects of multi extra dimensions may originate from the warped geometry with only one extra dimension. How to distinguish these two models and measure the number of the extra dimensions are interesting theoretical and experimental problems. More interestingly, the equivalent number N of the extra dimensions in the new models may be non-integral, which provides a new mechanism to escape the experimental constrains. Setting the fundamental scale $M_* \sim k \sim 1$TeV, the critical distance is given by $R_c = 2 \times 10^{\frac{32}{\alpha-2}-16}$mm. Then to satisfy the experimental constrain $R_c \lesssim 0.1$mm [16, 17], we only need $\alpha \gtrsim 4.18$ or $N \gtrsim 2.18$.

Furthermore, since the geometry is a slice of AdS$_5$, this new model may have a close relation to the AdS/CFT correspondence and has a holographic description [4, 18].

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